There are many directions in which this analysis might be extended, some of which are in progress. One extension would be to that of the power absorbed by an object in near fields, rather than plane wave fields. Another is to analyze the power absorbed by an oblate spheroid, which is a better model of some animals, such as the turtle and perhaps the rabbit, than the prolate spheroid. Also the oblate spheroid analysis could be applied to the power absorbed by some kinds of cells. Another application might be the calculation of power distribution in a physiological solution containing cells in a petri dish. The range of frequency for which the results are valid should be examined carefully, and the possibility of increasing the range of validity through higher order terms explored. Verification of the analysis should be made through careful measurements.

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Short Papers

Design Equations for an Interdigitated Directional Coupler

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***Abstract*—General design equations for an interdigitated directional coupler are derived. The design equations are written in terms of even- and odd-mode admittances for a pair of coupled lines which are identical to any pair of adjacent lines in the coupler. The calculated values of even- and odd-mode admittances can be translated into a physical configuration from published data on coupled lines.**

I. INTRODUCTION

Because of the advantages of broad-band, low loss, and tight coupling available in simple planar structures, the use of interdigitated structure as a directional coupling scheme has gained popularity among design engineers in recent years. Lange [1] first reported a 3-dB interdigitated microstrip hybrid in 1969. Later, in 1972, Waugh and LaCombe [2] constructed an "unfolded" version of a 3-dB Lange coupler and demonstrated that the performance is essentially the same as that of the Lange coupler, thus providing flexibility in geometrical layout of microstrip circuits. The extension of the interdigitated structure to a more loosely coupled directional coupler was recently reported by Miley [3].

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While the circuit has been fabricated and put into practical applications, unfortunately no general design method for interdigitated couplers has been published. Thus a designer is forced to use a trial-and-error approach to achieve his design objective. This short paper attempts to fill this gap by giving a theoretical treatment of the interdigitated structure. General design equations for a directional coupler are derived, which, in turn, can be translated into a physical configuration by using published data available in the literature.

II. ARRAY OF PARALLEL—COUPLED LINES

Consider first an array of k parallel-coupled TEM lines as shown in Fig. 1. It is assumed that the physical dimensions of each line are identical, so are the spacings between the lines. The total number of lines k is assumed to be even. Generally, there are couplings in existence between any pair of lines. However, for mathematical simplicity and practical consideration, only the couplings between the adjacent lines will be considered. The neglect of nonadjacent couplings is not a serious limitation in many practical applications, as pointed out by Matthaei [4]. Though not exact, the assumption of TEM mode provides good approximation for microstrip, as will be seen later from comparison of theoretical results and empirical data.

The current and voltage relation for such an array of transmission lines may be written as follows:

$$I_{ma} = -j \cot\theta \sum_{n=m-1}^{m+1} Y_{mn} V_{na} + j \csc\theta \sum_{n=m-1}^{m+1} Y_{mn} V_{nb} \quad (1)$$

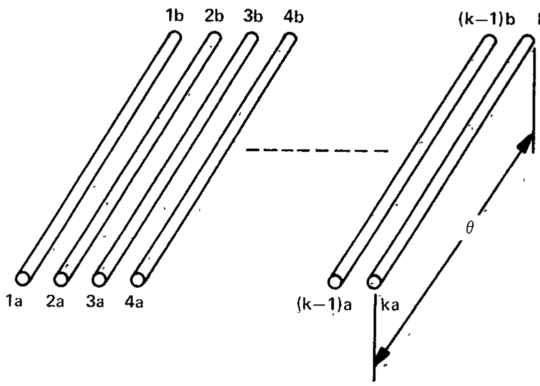

Fig. 1. Array of parallel-coupled lines.

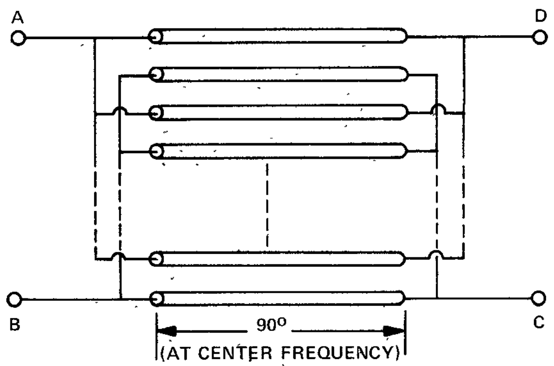

Fig. 2. Interdigitated directional coupler.

$$I_{mb} = j \csc\theta \sum_{n=m-1}^{m+1} Y_{mn}V_{na} - j \cot\theta \sum_{n=m-1}^{m+1} Y_{mn}V_{nb}, \quad m,n = 1,2,3,\cdots k \tag{2}$$

where I and V are the terminal current and voltage, respectively, at each of the $2k$ ports, and θ is the electrical length of the transmission lines. Y_{mn} can be expressed in terms of self and mutual capacitances and phase velocity [5]:

$$Y_{m(m+1)} = -v_p C_{m(m+1)} \tag{3}$$

and

$$Y_{mm} = v_p(C_{mo} + C_{(m-1)m} + C_{m(m+1)}) \tag{4}$$

where

$C_{m(m+1)}$ mutual capacitance per unit length between lines m and $m+1$;
C_{mo} capacitance per unit length between line m and ground;
v_p phase velocity.

From the previous assumptions that the lines as well as the spacings between them are identical, it is expected that the following relations exist:

$$C_{m(m+1)} = C_{12} \tag{5}$$

$$C_{mo} = \begin{cases} C_{1o}, & \text{if } m = 1,k \\ C_{2o}, & \text{if } m \neq 1,k. \end{cases} \tag{6}$$

Furthermore, it can be shown that C_{2o} is approximately related to C_{1o} and C_{12} according to (7):

$$C_{2o} \simeq C_{1o} - \frac{C_{1o}C_{12}}{C_{1o} + C_{12}}. \tag{7}$$

Substitution of (5)–(7) into (3) and (4) yields the following relations:

$$Y_{m(m+1)} = Y_{12} \tag{8}$$

$$Y_{mm} = \begin{cases} Y_{11}, & \text{if } m = 1,k \\ Y_{11} + Y_{12}^2/Y_{11}, & \text{if } m \neq 1,k. \end{cases} \tag{9}$$

To form an interdigitated structure, let the alternate terminals on both ends "a" and "b" be connected together, as shown in Fig. 2. The four ports of the resulting network will be assigned as ports A, B, C, and D, as also shown in Fig. 2. From (1) and (2), and (8) and (9), it follows that the terminal currents and voltages of the new four-port network are governed by the following relation:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} -jM\cot\theta & -jN\cot\theta & jN\csc\theta & jM\csc\theta \\ -jN\cot\theta & -jM\cot\theta & jM\csc\theta & jN\csc\theta \\ jN\csc\theta & jM\csc\theta & -jM\cot\theta & -jN\cot\theta \\ jM\csc\theta & jN\csc\theta & -jN\cot\theta & -jM\cot\theta \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix} \tag{10}$$

where

$$M = \frac{k}{2} Y_{11} + \left(\frac{k}{2} - 1\right) \frac{Y_{12}^2}{Y_{11}} \tag{11}$$

$$N = (k-1)\, Y_{12}. \tag{12}$$

From (3)–(9), it is easy to see that $M^2 > N^2$.

III. INTERDIGITATED DIRECTIONAL COUPLER

When the four-port network of Fig. 2 is used as a directional coupler, θ is chosen to be 90° at center frequency. From (10), the current–voltage relation at center frequency is reduced to:

$$\begin{bmatrix} I_A \\ I_B \\ I_C \\ I_D \end{bmatrix} = \begin{bmatrix} 0 & 0 & jN & jM \\ 0 & 0 & jM & jN \\ jN & jM & 0 & 0 \\ jM & jN & 0 & 0 \end{bmatrix} \begin{bmatrix} V_A \\ V_B \\ V_C \\ V_D \end{bmatrix}. \tag{13}$$

It can be shown that a four-port network can be used as a perfect directional coupler if its terminal current–voltage relation satisfies (13). For perfect match and perfect isolation, the terminated admittance at the four ports Y_o should follow the condition:

$$Y_o^2 = M^2 - N^2. \tag{14}$$

Assuming that port A is the input, the power coupling ratios are determined by the following equations:

$$\frac{P_B}{P_{\text{in}}} = \frac{N^2}{M^2} \tag{15}$$

$$\frac{P_C}{P_{\text{in}}} = 0 \tag{16}$$

$$\frac{P_D}{P_{\text{in}}} = \frac{M^2 - N^2}{M^2}. \tag{17}$$

Note that port C is perfectly isolated from port A. Substitution of

(11) and (12) into (14) and (15) provides the following design equations for the interdigitated directional coupler:

$$Y_o^2 = \left[\frac{k}{2} Y_{11} + \left(\frac{k}{2} - 1\right) \frac{Y_{12}^2}{Y_{11}}\right]^2 - [(k-1) Y_{12}]^2 \tag{18}$$

$$\frac{P_B}{P_{in}} = \left[\frac{2(k-1) Y_{11} Y_{12}}{k Y_{11}^2 + (k-2) Y_{12}^2}\right]^2. \tag{19}$$

It may be more informative to rewrite (18) and (19) in terms of even- and odd-mode admittances, Y_{oe} and Y_{oo}, of a pair of coupled lines which are identical to any adjacent pair of lines, without the presence of others, in the array. Since data of Y_{oe} and Y_{oo} for various coupled pairs of lines are available [6], the values of Y_{oe} and Y_{oo} can be readily translated into a physical configuration. From the definitions of Y_{oe} and Y_{oo}, it is easy to establish the following relations:

$$Y_{11} = \tfrac{1}{2}(Y_{oo} + Y_{oe}) \tag{20}$$

$$Y_{12} = -\tfrac{1}{2}(Y_{oo} - Y_{oe}). \tag{21}$$

Substituting (20) and (21) into (18) and (19), one obtains

$$Y_o^2 = \frac{[(k-1) Y_{oo}^2 + Y_{oo} Y_{oe}][(k-1) Y_{oe}^2 + Y_{oo} Y_{oe}]}{(Y_{oo} + Y_{oe})^2} \tag{22}$$

$$\frac{P_B}{P_{in}} = \left[\frac{(k-1) Y_{oo}^2 - (k-1) Y_{oe}^2}{(k-1) Y_{oo}^2 + 2 Y_{oo} Y_{oe} + (k-1) Y_{oe}^2}\right]^2. \tag{23}$$

When $k = 2$, i.e., for a pair of coupled line, (22) and (23) reduce to the familiar equations:

$$Y_o^2 = Y_{oo} Y_{oe} \tag{24}$$

$$\frac{P_B}{P_{in}} = \left(\frac{Y_{oo} - Y_{oe}}{Y_{oo} + Y_{oe}}\right)^2. \tag{25}$$

From (22) and (23), various interdigitated couplers using different number of lines for 50-Ω system, i.e., $Y_o = 1/50$, are calculated and listed in Table I.

It should be noted that the effects from bridging connections as well as from junction discontinuities have been neglected in the derivation of the design equations. As a result, there is a practical limitation on the number of lines to be used in the interdigitated coupler. In most applications, however, the choice of $k = 4$ seems to be quite satisfactory.

IV. COMPARISON WITH EMPIRICAL RESULTS

The performance of interdigitated couplers for two different ratios of power coupling, one for 3 dB and another 6.5 dB, has been published in the literature. Waugh and LaCombe [2] reported that a 4-finger 3-dB microstrip coupler can be constructed by using the following physical configuration:

dielectric constant:	$\epsilon_r \simeq 9.5$
substrate thickness:	$h = 0.025$ in
linewidth:	$w = 0.0028$ in
line spacing:	$s = 0.002$ in.

From interpolation in Bryant and Weiss' tables [6], it is estimated that, for this configuration, $Z_{oe} \simeq 170\ \Omega$ and $Z_{oo} \simeq 50\ \Omega$. The theoretical values for a 3-dB coupler with $k = 4$, as calculated in Table I, are $Z_{oe} = 176.2\ \Omega$ and $Z_{oo} = 52.61\ \Omega$, which agree quite well with the empirical results.

A second microstrip coupler, also of 4-finger interdigitated structure for 6.5-dB coupling, was constructed by Miley [3] according to the following data:

TABLE I

No. of Lines k	Power Coupling Ratio (dB)	Y_{oo} (℧)	Z_{oo} (Ω)	Y_{oe} (℧)	Z_{oe} (Ω)
2	3 dB	0.0483	20.71	0.00828	120.7
	6 dB	0.0346	28.87	0.0115	86.60
	10 dB	0.0277	36.04	0.0144	69.37
4	3 dB	0.0190	52.61	0.00568	176.2
	6 dB	0.0147	67.96	0.00702	142.5
	10 dB	0.0131	76.30	0.00845	118.3
6	3 dB	0.0121	82.55	0.00411	243.1
	6 dB	0.00951	105.1	0.00490	204.3
	10 dB	0.00819	122.1	0.00552	181.1

$\epsilon_r \simeq 9.5$
$h = 0.020$ in
$w = 0.003$ in
$s = 0.005$ in.

The even- and odd-mode impedances for this configuration are approximately equal to 135 and 65 Ω, respectively. Using (23), the power coupling is calculated to be about 6.12 dB. Thus, in this case, the calculated coupling and the measured result agree within about 0.2 dB when circuit loss is taken into consideration.

These two examples confirm the close agreement between the theoretical calculation and the measured performance.

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Intermodulation Distortion and Gain Compression in Varactor Frequency Converters

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***Abstract*—Using the nonlinear theory of Gardiner and Ghobrial [1] gain compression in varactor frequency converters is characterized and related to the distortion performance of the device. It is**

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